New Blockcipher Modes of Operation with Beyond the Birthday Bound Security

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Blockcipher Modes

Algorithms that provide

- privacy (encryption mode)
- authenticity (MAC)
- privacy and authenticity (AE mode)
- $\cdots$

based on blockciphers.

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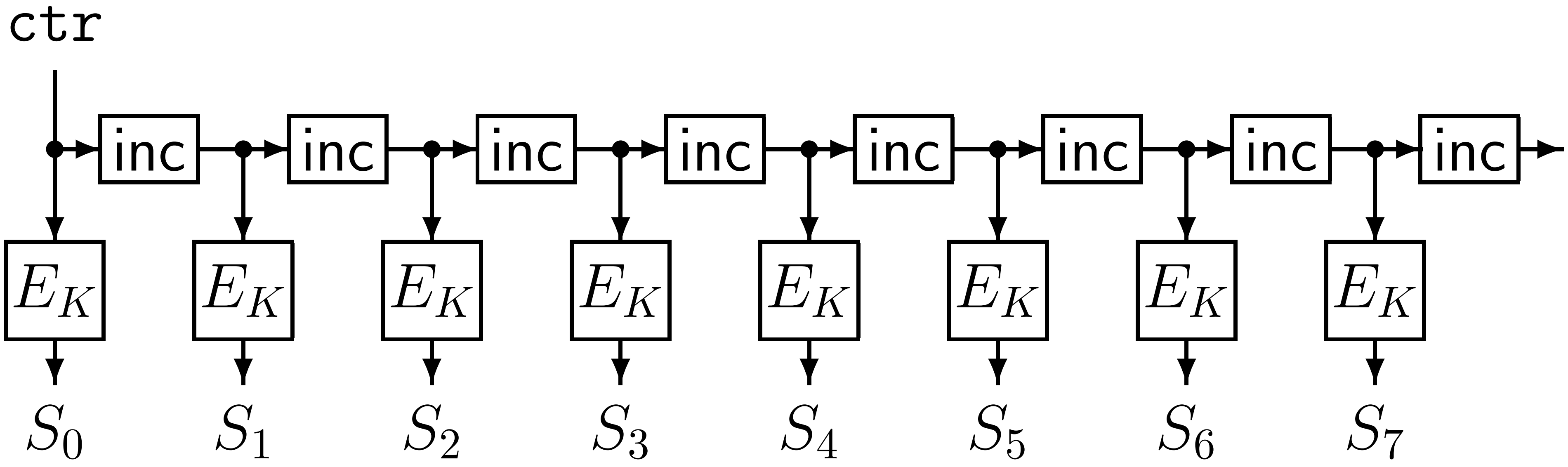
Known Encryption Modes

- ▷ CTR
- CBC
- OFB
- CFB
- ECB
- ...

CTR



- $S = (S_0, S_1, \ldots, S_7)$: keystream
- Encryption: $C = M \oplus S$
- Decryption: $M = C \oplus S$

Advantages of CTR

- provable security
- security proofs with the standard PRP assumption
- highly efficient
- single blockcipher key
- fully parallelizable
- allows precomputation of keystream
- allows random access

Security Definition

- "Indistinguishability from random strings"

  (Rogaway, Bellare, Black, Krovetz, '03)

- Scenario: Adaptive chosen plaintext attack

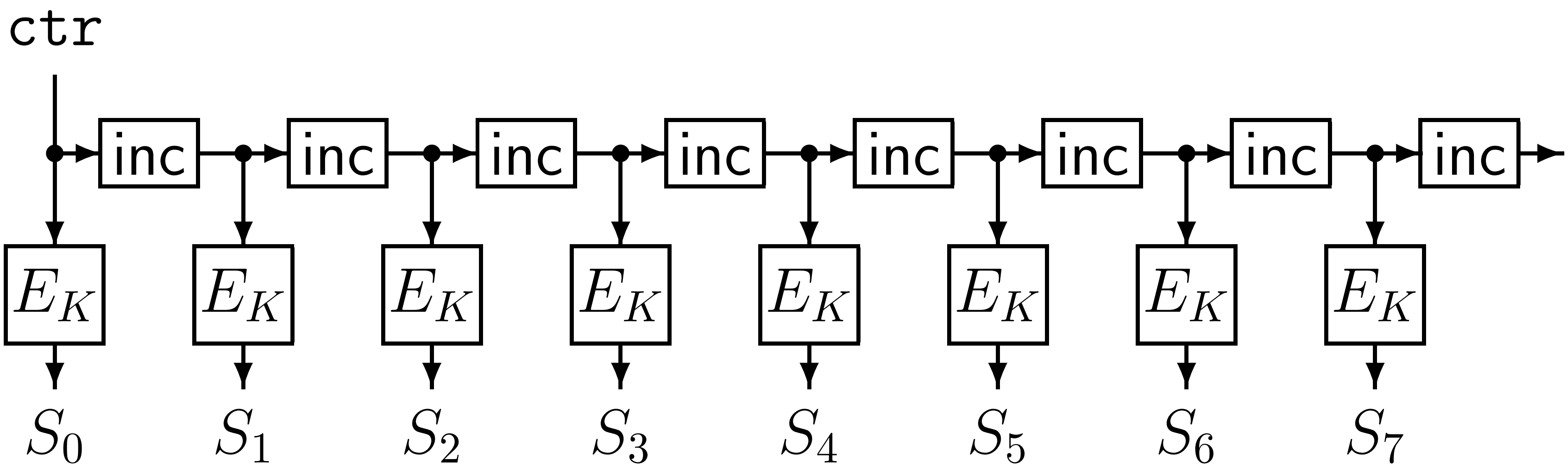
- Goal: To distinguish between

  – "real ciphertext"

  – "truly random string"

  (of the same length as ciphertext)

Keystream Generation Part of CTR



$S_i \neq S_j$ since $E_K(\cdot)$ is a permutation.

Keystream Generation Part of CTR

- If $S = (S_0, \ldots, S_{\sigma-1})$ is the keystream of CTR,

$$\Pr(S_i = S_j) = 0.$$

- If $S = (S_0, \ldots, S_{\sigma-1})$ is the truly random string,

$$\frac{0.3\sigma(\sigma - 1)}{2^n} \leq \Pr(S_i = S_j) \leq \frac{0.5\sigma(\sigma - 1)}{2^n}.$$

(n: length of S_i in bits, block size of E)

Keystream Generation Part of CTR

- For any A, $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CTR}}(A) \le \dfrac{0.5\sigma(\sigma-1)}{2^n}$.

Birthday Bound

- There exists A s.t. $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CTR}}(A) > \dfrac{0.3\sigma(\sigma-1)}{2^n}$.
  - ▷ A guesses "random string" if there is a collision.
  - ▷ Otherwise A guesses "ciphertext of CTR."

Security of CTR

CTR can **NOT** have beyond the birthday bound security (as long as $E_K(\cdot)$ is a permutation).

Our Work: New Encryption Mode

CENC $\cdots$ **C**ipher-based **ENC**ryption

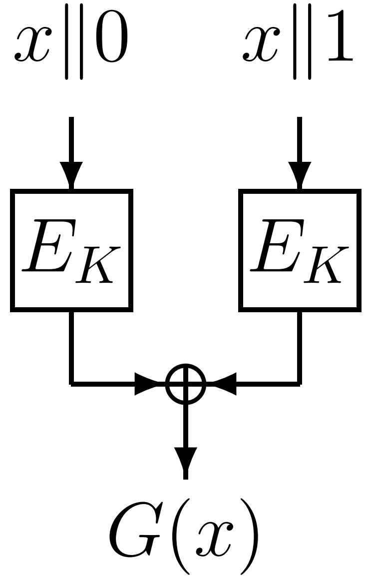
beyond the birthday bound security

without breaking advantages of CTR

The Basic Idea

- Convert $E_K(\cdot)$ into a function.
- $G_K(x) = E_K(x\|0) \oplus E_K(x\|1)$, $x \in \{0,1\}^{n-1}$

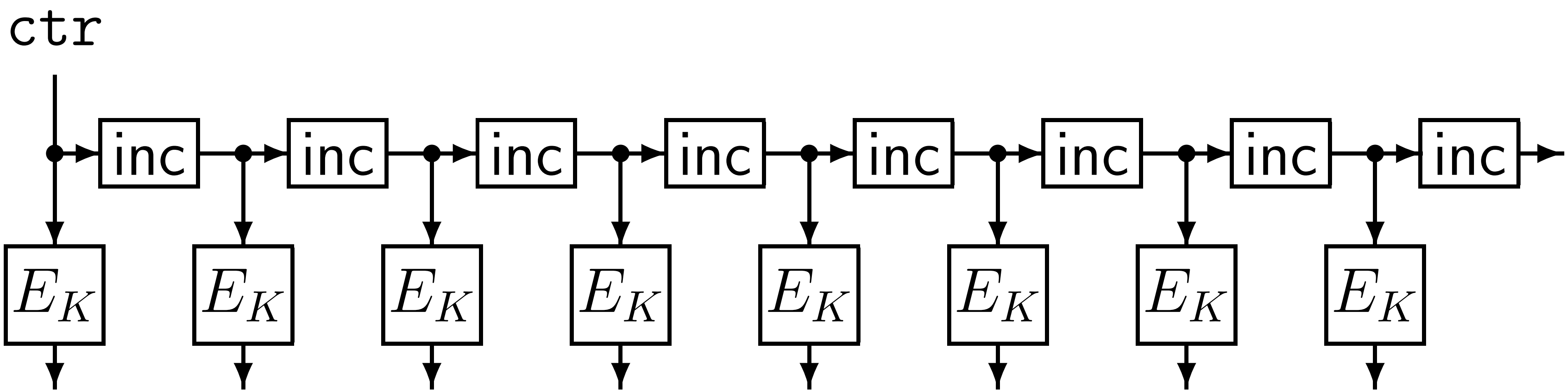
(Lucks '00, Bellare and Impagliazzo '99)



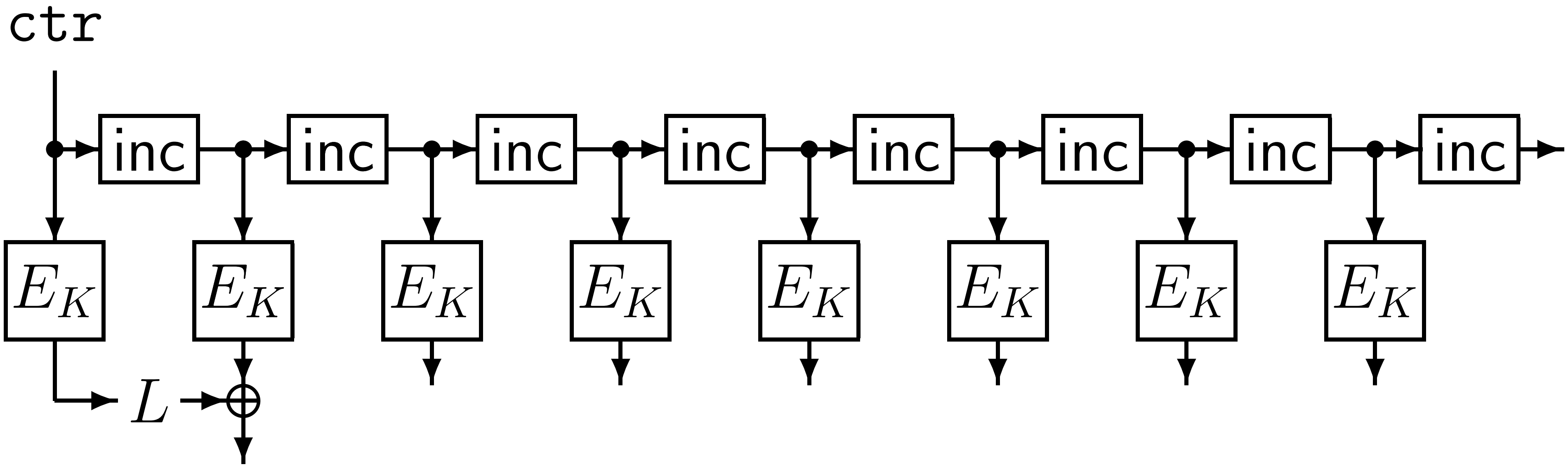
CENC Parameters

- Blockcipher $E : \{0,1\}^k \times \{0,1\}^n \to \{0,1\}^n$
- Nonce length: ℓ_{nonce} bits, $\ell_{\text{nonce}} < n$
- Frame width: w

Keystream Generation Part of CENC

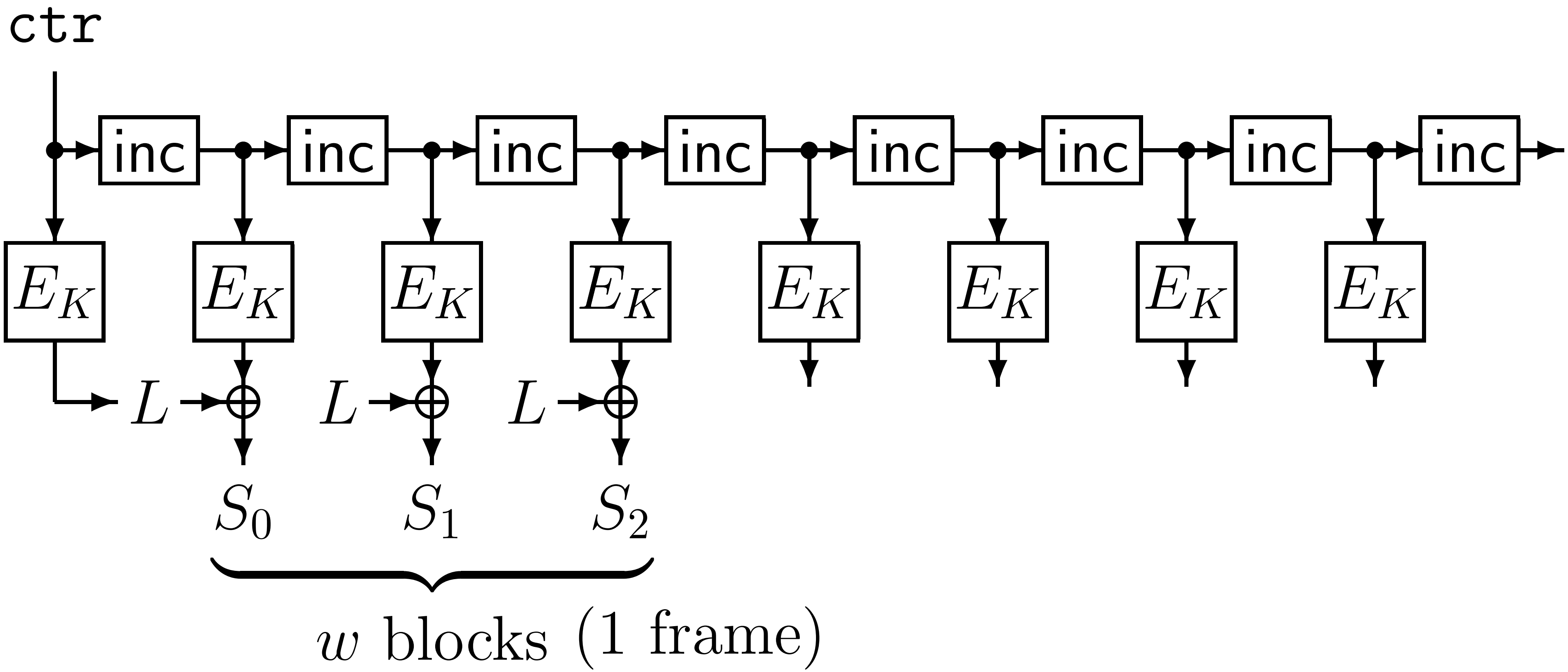


Keystream Generation Part of CENC



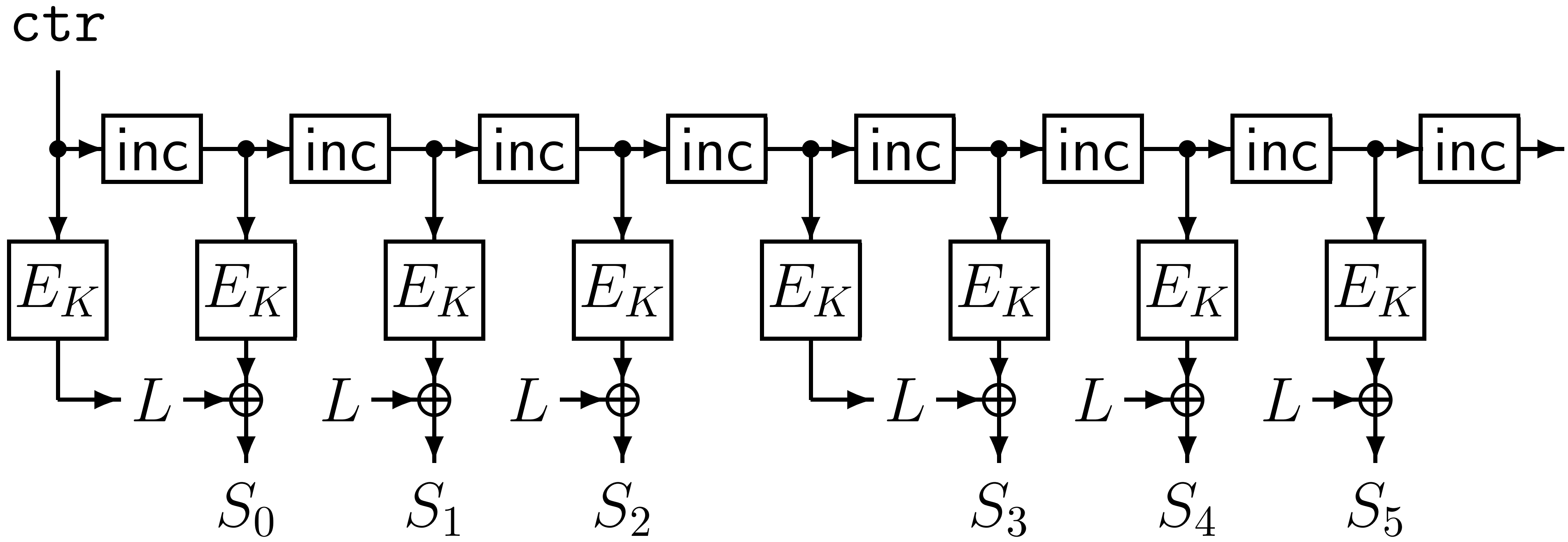
- L: mask

Keystream Generation Part of CENC

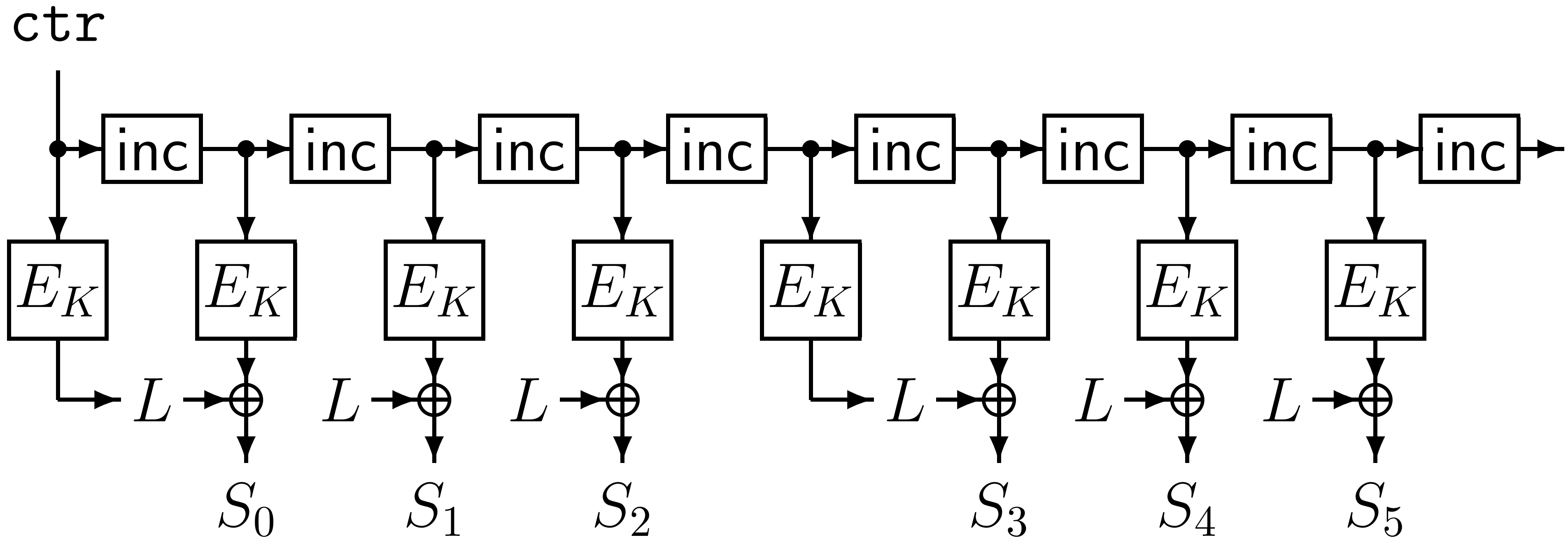


- w: frame width, default: $w = 2^8 = 256$

Keystream Generation Part of CENC

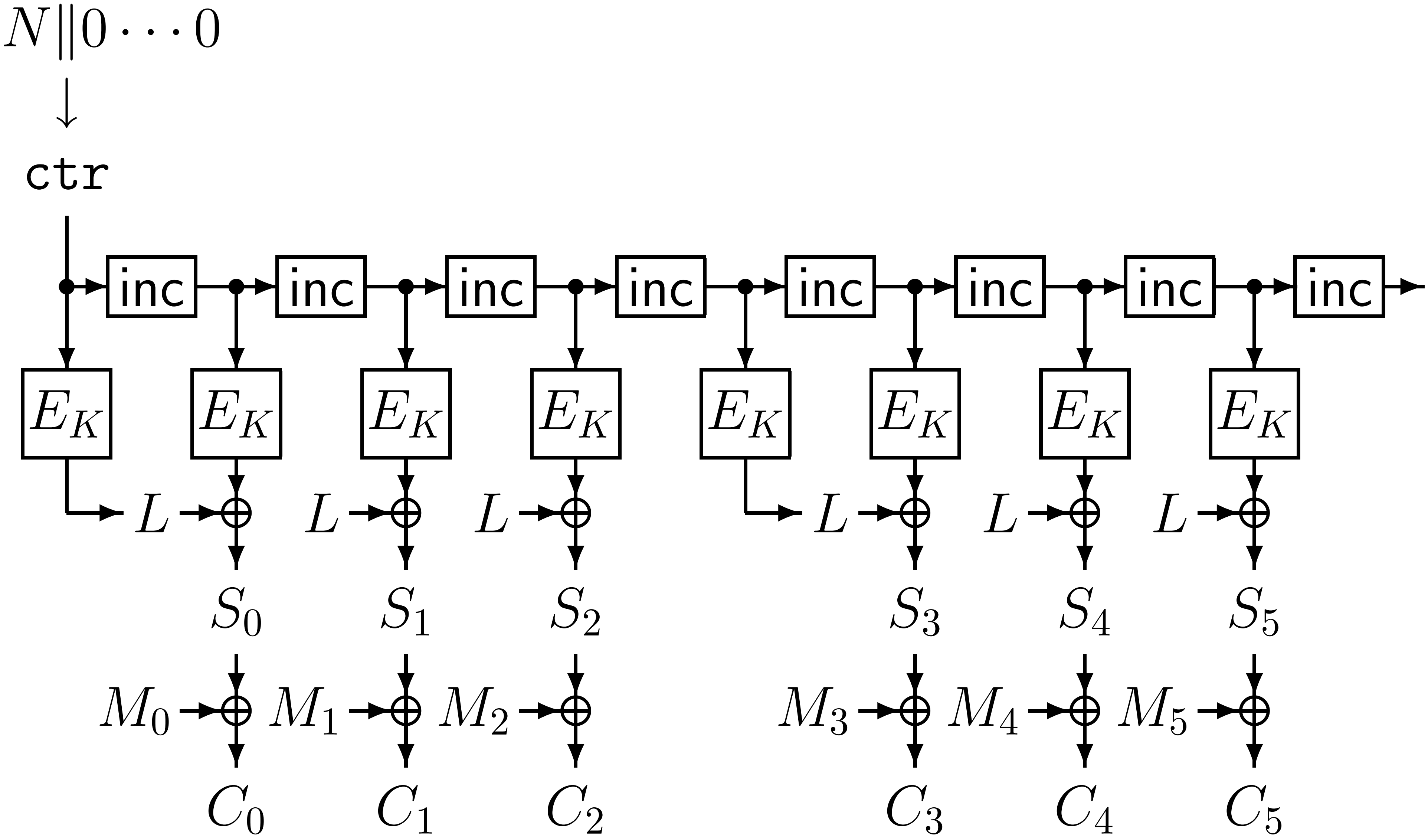


Keystream Generation Part of CENC



- N: Nonce, $\texttt{ctr} \leftarrow N \| 0 \cdots 0$
- default: $|N| = \ell_{\text{nonce}} = n/2$

Encryption Algorithm of CENC



Advantages of CENC

▷ provable security — beyond the birthday bound

• security proofs with the standard PRP assumption

▷ highly efficient — small cost

• single blockcipher key

• fully parallelizable

• allows precomputation of keystream

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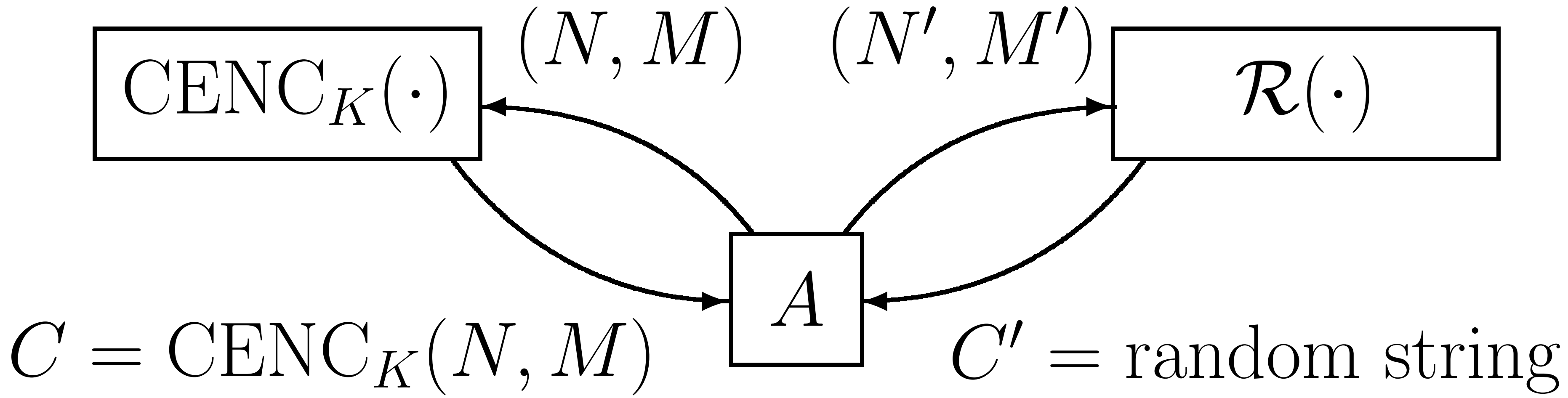
Indistinguishability from Random Strings

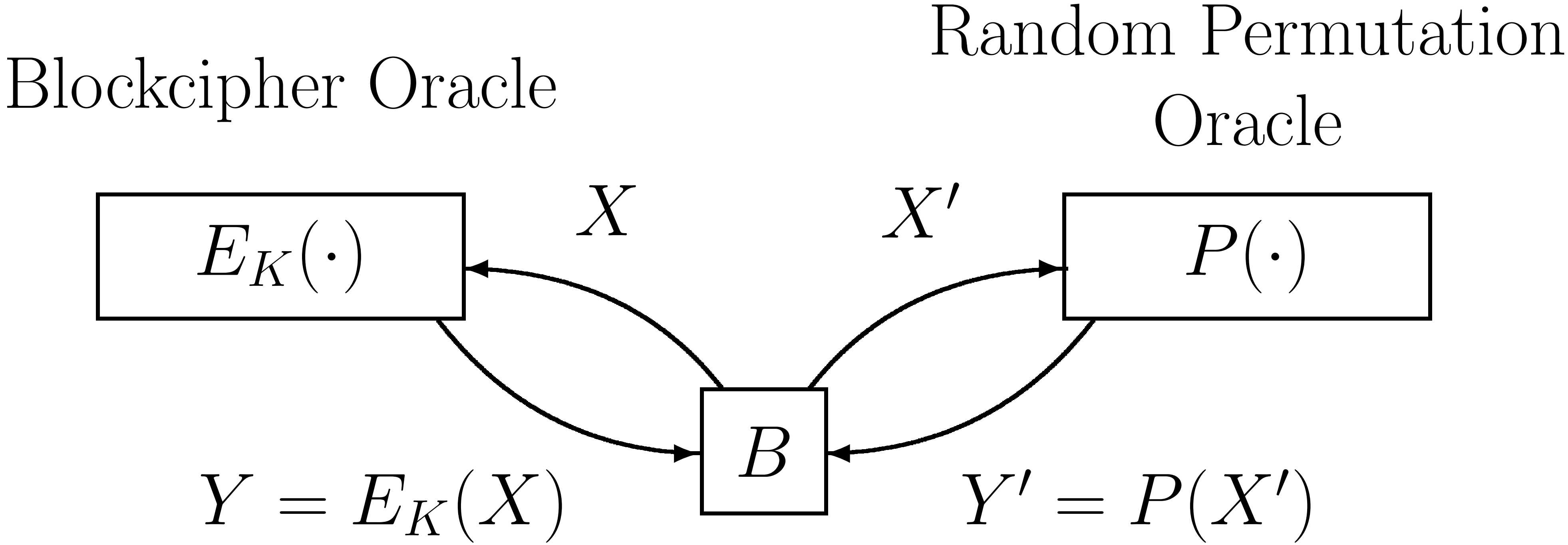
Encryption Oracle | Random String Oracle



A must not repeat nonce

$$\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CENC}}(A) \stackrel{\mathrm{def}}{=} \left| \Pr_K(A^{\mathrm{CENC}_K(\cdot,\cdot)} = 1) - \Pr_{\mathcal{R}}(A^{\mathcal{R}(\cdot,\cdot)} = 1) \right|$$

Security Definition for E (PRP, LR '88)



$$\mathbf{Adv}_E^{\mathrm{prp}}(B) \stackrel{\mathrm{def}}{=} \left| \Pr_K(B^{E_K(\cdot)} = 1) - \Pr_P(B^{P(\cdot)} = 1) \right|$$

Theorem. If there exists A against CENC such that:

- at most q queries, and
- at most σ blocks,

then there exists B against E such that:

- $\mathit{time}(B) = \mathit{time}(A) + O(n\hat{\sigma}w)$,
- at most $(w+1)\hat{\sigma}/w$ queries, and
- $\mathbf{Adv}_E^{\mathrm{prp}}(B) \geq \mathbf{Adv}_{\mathrm{CENC}}^{\mathrm{priv}}(A) - \dfrac{w\hat{\sigma}^3}{2^{2n-3}} - \dfrac{w\hat{\sigma}}{2^n}$,

where $\hat{\sigma} = \sigma + qw$.

Interpretation

- CENC is secure up to 2^{82} blocks (AES, $w = 2^8$).

▷ CTR is secure up to 2^{64} blocks.

If we encrypt $\sigma \le 2^{n/2}$ blocks,

- $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CENC}}(A) \le \dfrac{w\hat{\sigma}^3}{2^{2n-3}} + \dfrac{w\hat{\sigma}}{2^n} \le \dfrac{2w\hat{\sigma}}{2^n}$

▷ $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CTR}}(A) \le \dfrac{0.5\sigma^2}{2^n}$ (w: constant, $\hat{\sigma} \approx \sigma$)

Cost for the Security Improvement

$w + 1$ blockcipher calls for w blocks of keystream

- 257 calls to encrypt 256 blocks (Default: $w = 2^8$)
  - ▷ The cost is $1/257 = 0.4\%$ compared to CTR.
- 1 frame is w blocks, which is 4KBytes.
  - ▷ 99.9% of the Internet traffic is less than 1.5KBytes.
  - ▷ The cost is *one* blockcipher call compared to CTR.

New Authenticated-Encryption Mode

CHM $\cdots$ **C**ENC with **H**ash-based **M**AC

- CENC for privacy.
- Hash-based MAC (Wegman-Carter MAC) for authenticity.
- Beyond the birthday bound security.
- Similar to GCM by McGrew & Viega.

Open Question

▷ The security bound of CTR is tight.

- $\forall A$, $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CTR}}(A) \le 0.5\sigma(\sigma-1)/2^n$
- $\exists A$, $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CTR}}(A) > 0.3\sigma(\sigma-1)/2^n$

$$\forall A,\ \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CENC}}(A) \le w\hat{\sigma}^3/2^{2n-3} + w\hat{\sigma}/2^n$$

▷ Improve the security bound

▷ Attack with $\mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CENC}}(A) > \Omega(w\hat{\sigma}^3/2^{2n-3} + w\hat{\sigma}/2^n)$

Conjecture

The security bound can be improved.

$$\forall A,\ \mathbf{Adv}^{\mathrm{priv}}_{\mathrm{CENC}}(A) \le O(w\hat{\sigma}/2^n)$$

Conclusion

- New encryption mode, CENC
- New AE mode, CHM
- beyond the birthday bound security

Questions?

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