Exact Security Analysis of Hash-then-Mask Type Probabilistic MAC Constructions

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27th September, 2016

Outline of the talk

1. Message Authentication Code.
2. HtM Construction.
3. Contributions.
4. Conclusion

MAC (Stateless and Deterministic): The Popular Story

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Unforgeability

- Adversary asks for tags for queries of his choice.
- Goal is to generate any fresh, valid (message, tag) pair.

Security Requirement: It should be HARD

MAC (Stateful or Probabilistic): The Popular Story

• Alice sends a message M, an auxiliary variable IV with a tag $T = MAC_K(M, IV)$ corresponding to the message M and IV to Bob.

• Data Integrity: Bob verifies the sender and the message by computing $VER_K(M, IV, T) = 1$.

Stateful MAC : When IV is a counter / nonce. (e.g XMACC, PCS)
Probabilistic MAC : When IV is random. (e.g XMACR, EHtM)

Unforgeability

- Adversary asks for T for queries M (Signing Query).
- Adversary asks fresh (M, IV, T) triplet and obtains 1 or 0. Succeed if the response is 1 (Verification Query).

Security: Should be HARD to obtain response 1

Pseudo Random Function (PRF)

PRF

Keyed function which is indistinguishable from a Random Function (RF)

Indistinguishability

- Responses of adversary queries are given either using the function or a RF.
- Goal is to distinguish the function from a RF.

Security Requirement: It should be HARD

Universal and AXU-Hash

Universal Hash

H is a n bit Universal Hash, if for all distinct values, the collision probability of H is negligible.

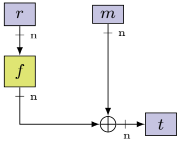
Almost-XOR-Universal Hash

H is a n bit AXU Hash, if for all distinct values x, x' and for all y, $\Pr[H(x) \oplus H(x') = y]$ is negligible.

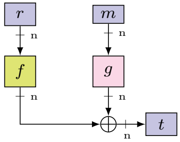
Existing Result on Probablistic MAC

Candidate	Construction	Rand	Eff.	Bound
XMACR[BGR'95]	$(r, H(m) \oplus f(r))$	n	$1H_{\mathrm{xu}}, 1F[n,n]$	$O(\frac{q^2}{2^n} + q_v\epsilon)$
MACRX$_3$[BGK'99]	$(r_1, r_2, r_3, \bigoplus_{i=1}^{3} f(r_i) \oplus H(m))$	$3n$	$1H_{\mathrm{xu}}, 3F[n,n]$	$O(\frac{q^3}{2^{3n}} + q_v\epsilon)$
RMAC[JJV'02]	$(r, f_2^r(\mathrm{CBC}_{f_1}(m))$	n	$(\ell+1)P[n]$	$O(\frac{\ell(q+q_v)}{2^n})$
FRMAC[JJ'04]	$(r, \pi_r(H(m)))$	n	$1H_{\mathrm{u}}, 1P[n,n]$	$O(\ell(q+q_v)\epsilon)$
RWMAC[M'10]	$(r, g(r, H(m)))$	n	$1H_{\mathrm{u}}, 1F[2n,n]$	$O(\frac{q^2\epsilon}{2^n} + q_v\epsilon)$
EHtM[M'10]	$(r, f(r) \oplus g(r \oplus H(m))$	n	$1H_{\mathrm{xu}}, 2F[n,n]$	$O(\frac{q^3\epsilon}{2^n} + q_v\epsilon)$

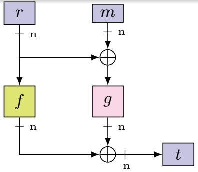
HtM: Probabilistic MAC



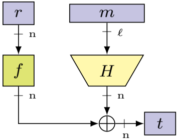
C1 : $t := f(r) \oplus m$



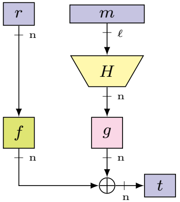
C3 : $t := f(r) \oplus g(m)$



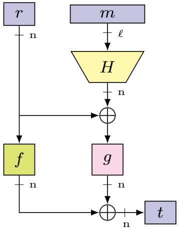
C5 : $t := f(r) \oplus g(r \oplus m)$



C2 : $t := f(r) \oplus H(m)$



C4 : $t := f(r) \oplus g(H(m))$



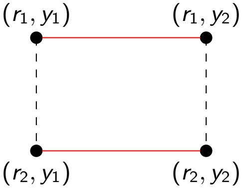
C6 : $t := f(r) \oplus g(r \oplus H(m))$

Our Contribution

1. Tight PRF, pPRF and MAC Security Analysis of Different Types of HtM Constructions.
2. An Impossibility Result on Probabilistic MAC:
   Unlike deterministic MAC, in probabilistic MAC, there is no such ideal system, indistinguishable to which, ensures forging advantage.

	C1	C2	C3	C4	C5	C6
PRF	X	X	X	X	X	$\Theta(2^{n/2})$
pPRF	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{3n}{4}})$	$\Theta(2^{\frac{3n}{4}})$
MAC	X	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{n}{2}})$	$\Theta(2^{\frac{2n}{3}})$	$\Theta(2^{\frac{3n}{4}})$

PRF Attack Idea of C1,C2,C3,C4



$$\boxed{\mathrm{SUM}_{\mathrm{f,g}}(r, y) = f(r) \oplus g(y)}$$

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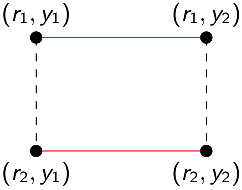

$$\text{SUM}_{\mathrm{f,g}}(r, y) = f(r) \oplus g(y)$$

Alternating Cycle (Alt-Cycle)

- For an Alt-Cycle C, $\sum_{i=1}^{4} \text{SUM}^{C}_{\mathrm{f,g}}(r_i, y_i) = 0$ (distinguishing event)
- For C1, C2 : g is identity function.
- For C1, C3 : y is m; For C2, C4 : y is $H(m)$; For C5 : y is $r + m$

PRF Attack Idea of C5 and C6

Attack Algorithm C5 : $f(r) \oplus g(r \oplus m)$

- Choose $(r_1, m_1), (r_2, m_2)$ s.t $r_1 + m_1 = r_2 + m_2$
- Query Phase :
  $t_1 \leftarrow (r_1, m_1), t_2 \leftarrow (r_2, m_2), t_3 \leftarrow (r_1, m_2), t_4 \leftarrow (r_2, m_1)$
- Distinguishing Event : If $\bigoplus_{i=1}^{4} t_i = 0$, return 1.

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Attack Algorithm C6 : $f(r) \oplus g(r \oplus H(m))$

- Query Phase :
  $t_1 \leftarrow (r, m_1), t_2 \leftarrow (r, m_2), \ldots, t_{2^{n/2}} \leftarrow (r, m_{2^{n/2}})$
- If $H(m_i) = H(m_j)$, query $t_i' \leftarrow (r', m_i), t_j' \leftarrow (r', m_j)$, output 1 if $t_i' = t_j'$
- Else, collision in g.

Probabilistic PRF (pPRF)

Definition and Security Game

Keyed function that takes two inputs (r, M) is indistinguishable from RF

- Adversary can only query the oracle with M.
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pPRF Attack Algorithm of C1 : $f(r) \oplus m$

- Query Phase : $t_1 \leftarrow m_1, t_2 \leftarrow m_1, \ldots, t_{2^{n/2}} \leftarrow m_1$
- W.h.p $\exists i, j \in \{1, 2, \ldots, 2^{n/2}\}$ s.t $r_i = r_j$
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pPRF Attack for C2, C3, C4 is same as that of C1

pPRF Attack Idea of C5

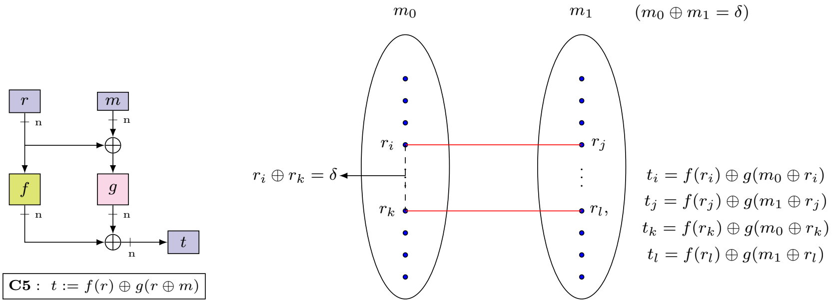
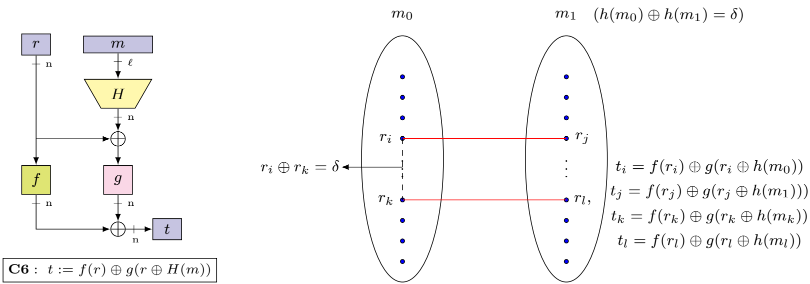


Figure 0.1: Distinguishing Event : If $t_i \oplus t_j \oplus t_k \oplus t_l = 0$, output 1.

pPRF Attack Idea of C6



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Forging Idea of C1-C6

Forging C1 : $f(r) \oplus m$

- Query Phase : $t \leftarrow (r, m)$.
- Forge : $(r, m \oplus \delta, t \oplus \delta)$.

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- W.h.p $i, j \in \{1, 2, \ldots, 2^{n/2}\}$ such that $r_i = r_j$. It leaks $H(m_i) \oplus H(m_j) = \delta$.
- Query $t \leftarrow (r, m_i)$.
- Forge : $(r, m_j, t \oplus \delta)$.

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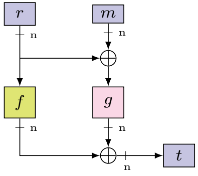
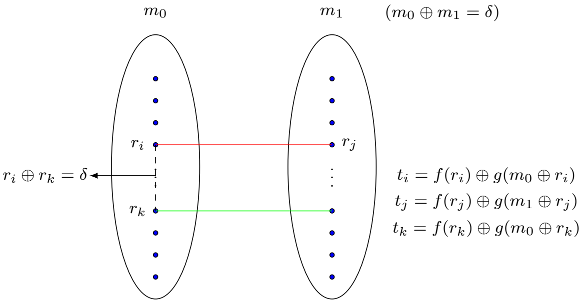
- Forge : $(r, m_j, t \oplus \delta)$.

Forging attack of C3, C4 is same as that of C2

Forging Idea of C5



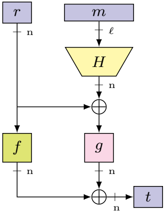
C5 : $t := f(r) \oplus g(r \oplus m)$



Forge: $(r_k, m_1, t_i \oplus t_j \oplus t_k)$

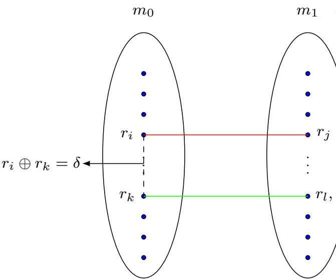
$(\because t_i \oplus t_j \oplus t_k = f(r_k) \oplus g(m_1 \oplus r_k))$

Forging Idea of C6



C6 : $t := f(r) \oplus g(r \oplus H(m))$

$(h(m_0) \oplus h(m_1) = \delta)$



$t_i = f(r_i) \oplus g(h(m_0) \oplus r_i)$
$t_j = f(r_j) \oplus g(h(m_1) \oplus r_j)$
$t_k = f(r_k) \oplus g(h(m_0) \oplus r_k)$

Forge: $(r_k, m_1, t_i \oplus t_j \oplus t_k)$

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Alternating Cycle

A transcript $\tau := \{(x_1, y_1), (x_2, y_2), \ldots, (x_q, y_q)\}$ has an alternating-cycle in τ of length k (k is even and ≥ 2), if we have k pairwise distinct indices $i_1, i_2, \ldots, i_k$ such that

$x_{i_1} = x_{i_2}, y_{i_2} = y_{i_3}, x_{i_3} = x_{i_4}, \ldots, x_{i_{k1}} = x_{i_k}, y_{i_k} = y_{i_1}$.

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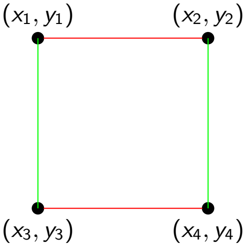

Figure: Alternating Cycle of length 4. Red line indicates first coordinate matches. Green line indicates second coordinates matches

Benes Butterfly Result

Theorem (Benes-Butterfly (AV'96))

Let f and g be two n-bit independent and uniformly distributed random functions. Let us consider a transcript $\tau = \{(x_i, y_i, t_i)_{1 \le i \le q}\}$ which does not contain any alternating cycle. Then

$$\Pr[f(x_i) \oplus g(y_i) = t_i, 1 \le i \le q] = \frac{1}{2^{nq}}.$$

Proof Sketch : If there is no alternating cycle in $\tau = \{(x_i, y_i)_{1 \le i \le q}\}$ then from each of q many equations, we get at least one uniform random variable

pPRF Advantage of C5 and C6

Theorem

$$\mathbf{Adv}^{\mathrm{pprf}}_{\mathrm{C5/C6}}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{f_{k_1}}(q,t) + \mathbf{Adv}^{\mathrm{prf}}_{f_{k_2}}(q,t) + \frac{q^4}{2^{3n}}.$$

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- Bad Transcript : Alternating cycle on $(r, r \oplus m)/(r, r \oplus h(m))$.

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SUF Advantage of C5 and C6

Theorem (SUF Advantage of C5)

$$\mathbf{Adv}^{\mathrm{suf}}_{C_5}(q, q', t) \leq \mathbf{Adv}^{\mathrm{prf}}_{f_{k_1}}(q + q', t) + \mathbf{Adv}^{\mathrm{prf}}_{f_{k_2}}(q + q', t) + \frac{q^3}{2^{2n}} + \frac{q'}{2^n}.$$

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• Bad Transcript : Alternating cycle on $(r, r \oplus m)$ after making signing and verfication queries.

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- Good Transcript $\Rightarrow$ No Alternating Cycle in the transcript.
- Probability of Bad Transcript : $\frac{q^3}{2^{2n}}$.

Proof Idea of SUF Advantage of C5 and C6

Theorem (SUF Advantage of C6)

$$\mathbf{Adv}^{\mathrm{suf}}_{\mathrm{C6}}(q, q', \ell, t) \leq \mathbf{Adv}^{\mathrm{prf}}_{f_{k_1}}(q+q', t') + \mathbf{Adv}^{\mathrm{prf}}_{f_{k_2}}(q+q', t') + \frac{q^4}{2^{3n}} + \frac{10q'}{2^n},$$

where $t = t' + O(qT_h)$

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- Good Transcript $\Rightarrow$ No Alternating cycle.
- Probability of Bad Transcript : $\frac{q^4}{2^{3n}}$ as (we need one more point)

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- Tight Security Analysis of EHtM.
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Thank You