More on the Automatic Search for Differential Trails in NORX (Work in Progress)

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Outline

Resistance of NORX Against DC

- Designers give bounds against DC [AJN14]
- Use SAT-solver; memory is exhausted for more rounds
- Bounds based on best trails for up to $F^{2.0}$ rounds

W	32				64			
Scenario	init_N	$\text{init}_{N,K}$	rate	full	init_N	$\text{init}_{N,K}$	rate	full
$F^{0.5}$	6	2	2	0	6	2	2	0
$F^{1.0}$	(60)	22	10	2	(53)	22	12	2
$F^{1.5}$	(60)	(40)	(31)	12	(53)	(35)	(27)	12
$F^{2.0}$	(61)	(45)	(34)	(27)	(51)	(37)	(30)	(23)

Scenario: modify the nonce (init_N), nonce + key ($\text{init}_{N,K}$), rate words (rate), full state (full).

Motivation

Research Goal

Provide tighter bounds than the ones reported by [AJN14]

Disclaimer: Work-in-progress; preliminary results.

Our Contributions

1. New algorithm for finding optimal trails – **Best Search (BS)**
2. Heuristic version of BS – **Heuristic Search (HS)**
3. New (sub-optimal) trails on up to $F^{2.0}$ rounds with HS

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Outline

Overview of NORX

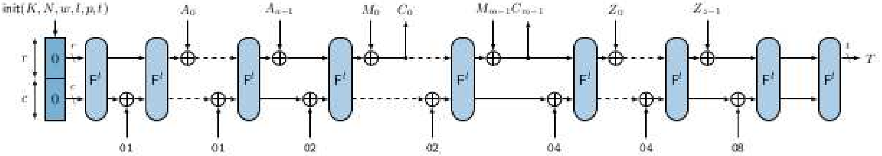
- Word size: $W \in \{32, 64\}$ bits
- Rounds: $1 \leq R \leq 63$
- Parallelism: $0 \leq D \leq 255$
- Tag size: $|A| \leq 10W$

NORXW-R-D	Nonce (2W)	Key (4W)	Tag (4W)	Classification
NORX64-4-1	128	256	256	Standard
NORX32-4-1	64	128	128	Standard
NORX64-6-1	128	256	256	High security
NORX32-6-1	64	128	128	High security
NORX64-4-4	128	256	256	High throughput

Credits: Philipp Jovanovic, ESORICS 2014.

NORX = NO(T A)RX

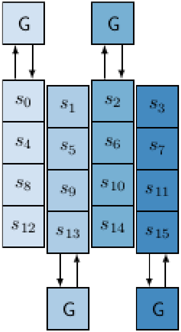
- Sponge structure based on the monkeyDuplex construction
- LRX permutation: $\oplus, \ggg, \wedge, \ll 1$
- Non-linear component: $H(x, y) = (x \oplus y) \oplus ((x \wedge y) \ll 1)$
- No S-box; No modular addition

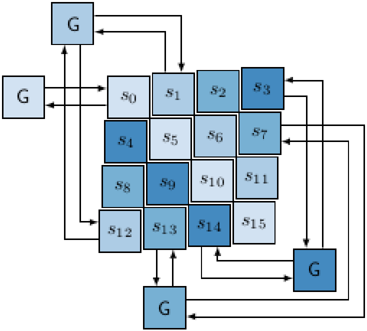


Credits: NORX Specification `https://norx.io/`

PERMUTATION F

F is composed of G-**circuit** (next slide) applied in parallel first to each of the 4 columns of the state followed by an application to each of the 4 diagonals of the state: $F = F_{\text{dia}} \circ F_{\text{col}}$

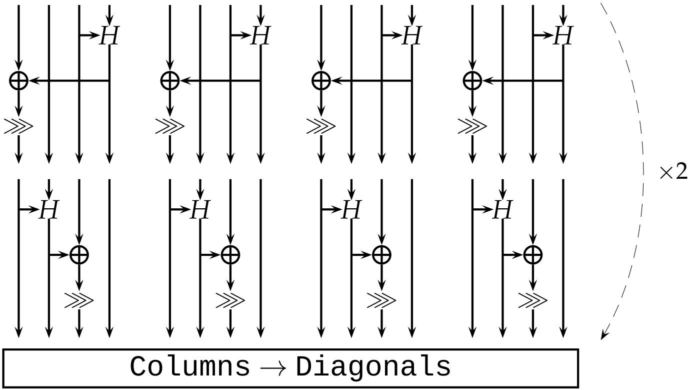




Credits: NORX Specification `https://norx.io/`

HALF-ROUND: $F^{0.5} = F_{\text{col}}$

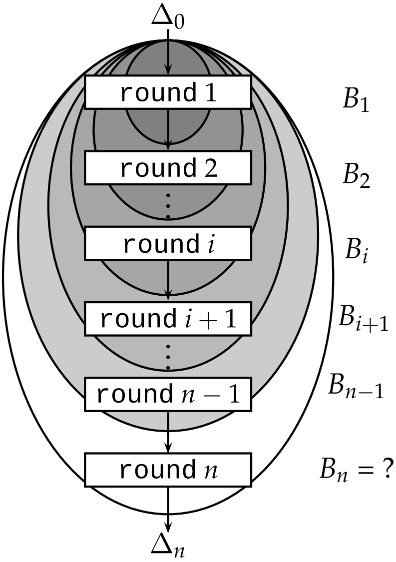
4 parallel app. of the G-circuit, where $H(x, y) = (x \oplus y) \oplus ((x \wedge y) \ll 1)$.



Note: Initialization = F^8; Data processing = F^4.

Outline

Matsui's Algorithm



Input: best p for $n-1$ rounds:

$B_1, B_2, \ldots, B_{n-1}; \bar{B}_n$

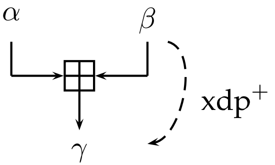
Output: best p for n rounds:

B_n

if $P_1 P_2 \ldots P_i B_{n-i} \geq \bar{B}_n$: $i \leftarrow i+1$

if $i = n$: $B_n \leftarrow \bar{B}_n$

MATSUI'S ALGORITHM FOR ARX [BVLC16]



$$2^{-\mathrm{hw}(\neg \mathrm{eq}(\alpha,\beta,\gamma) \wedge \mathrm{mask}_{n-1})}$$ [LM01]

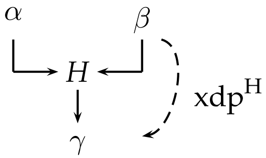
Proposition (Monotonicity of xdp^+)

xdp^+ *is monotonously decreasing with the word size* w *of* α, β, γ*:*

$$\tilde{p}_1 \geq \tilde{p}_2 \ldots \geq \tilde{p}_{w-1} \geq \tilde{p}_w = \mathrm{xdp}^+(\alpha, \beta \to \gamma) \ ,$$

where $\tilde{p}_i = \mathrm{xdp}^+(\alpha[i-1:0], \beta[i-1:0] \to \gamma[i-1:0]) : \ w \geq i \geq 1$ *, is the probability of the partial differential composed of the i LS bits of* α, β, γ.

MONOTONICITY OF xdp^{H}



$$H(x, y) = (x \oplus y) \oplus ((x \wedge y) \ll 1)$$

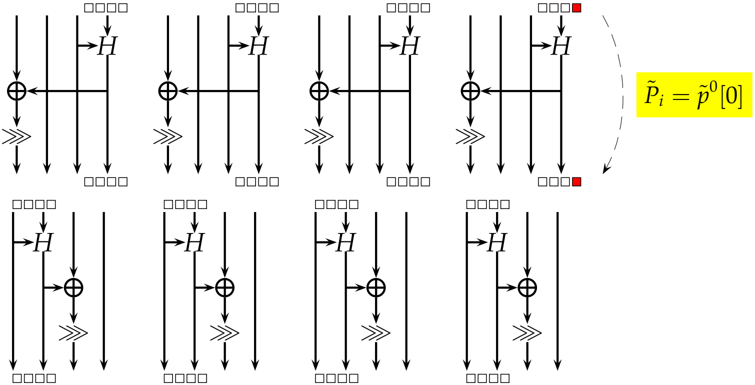
The Differential Probability of H [AJN14]

$$\text{xdp}^{\text{H}}(\alpha, \beta \to \gamma) = \begin{cases} 2^{-\text{hw}((\alpha \vee \beta) \ll 1)} & \iff (\alpha \oplus \beta \oplus \gamma) \wedge (\neg((\alpha \vee \beta) \ll 1)) = 0 \\ 0, & \text{otherwise} \end{cases}$$

Proposition (Monotonicity of xdp^{H})

xdp^{H} *is monotonously decreasing with the word size of* α, β, γ.

Best Search (BS): Rounds i and $i+1$



if $P_1 P_2 \ldots \tilde{P}_i B_{n-i} \geq \bar{B}_n$ assign next bit

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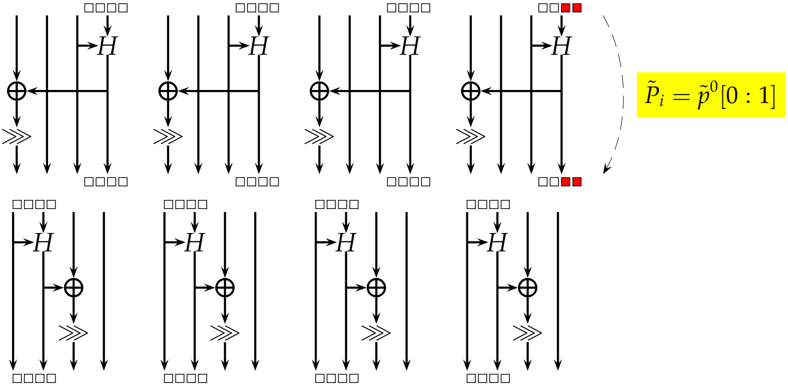

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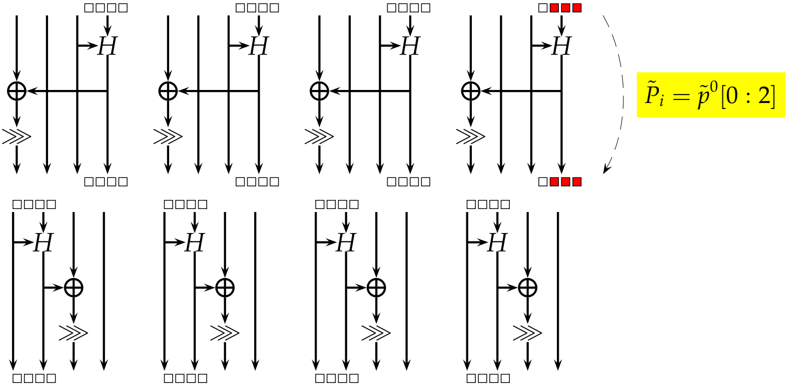

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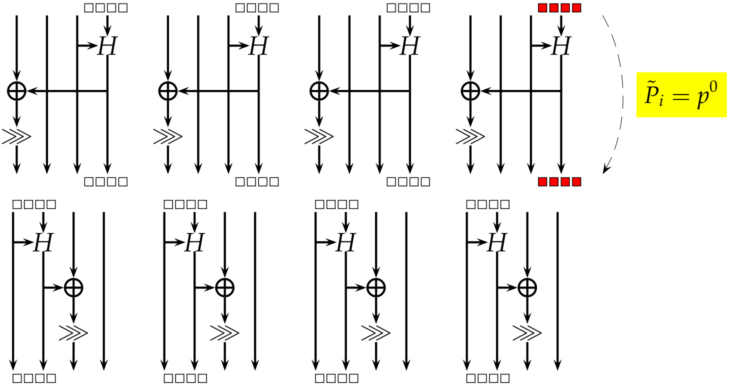

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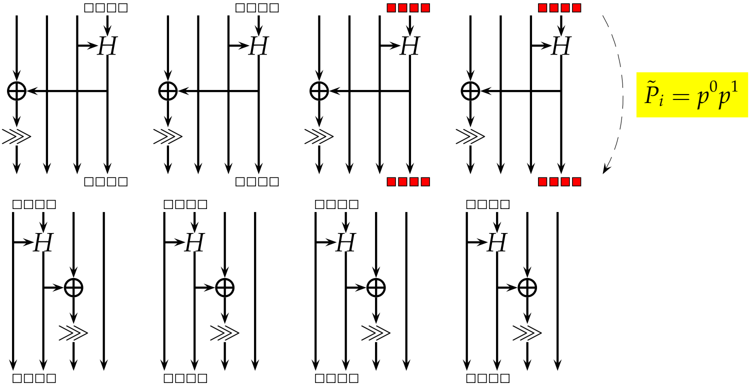

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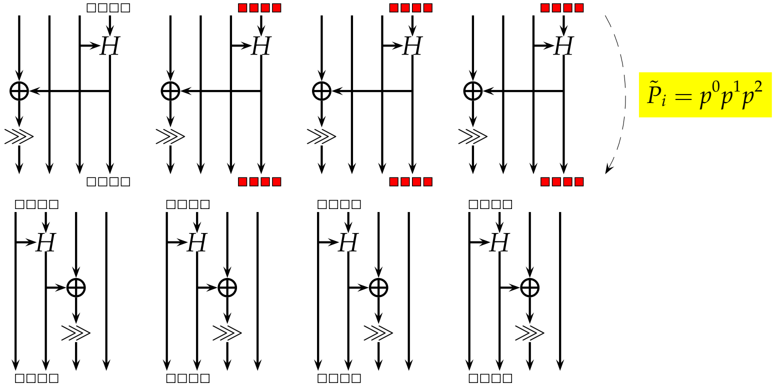

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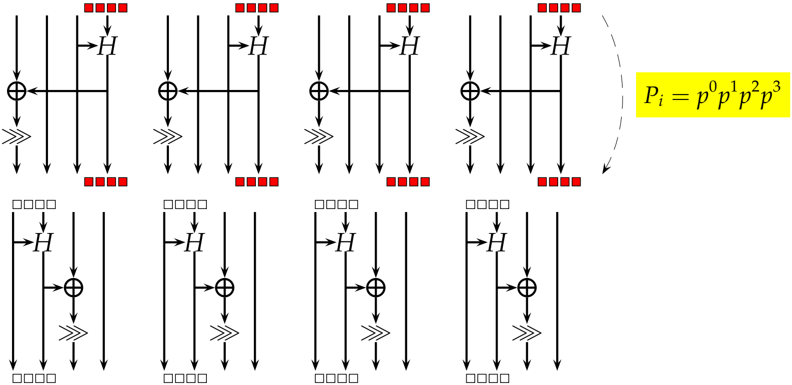

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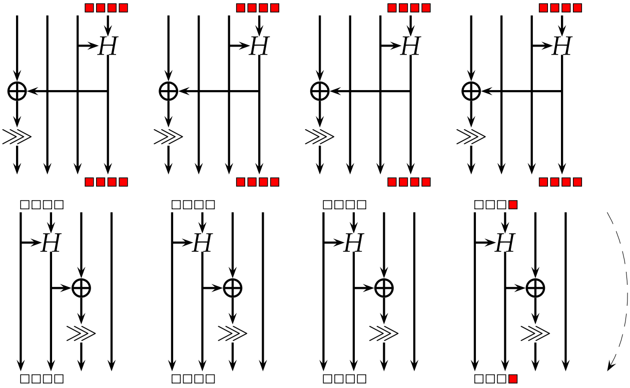
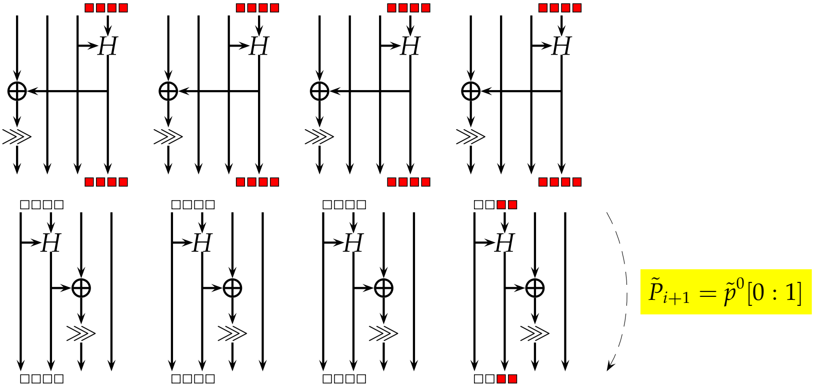


$\tilde{P}_{i+1} = \tilde{p}^{0}[0]$

`if` $P_1 P_2 \ldots P_i \tilde{P}_{i+1} B_{n-i-1} \geq \bar{B}_n$ `assign next bit`

BEST SEARCH (BS): ROUNDS i AND $i+1$



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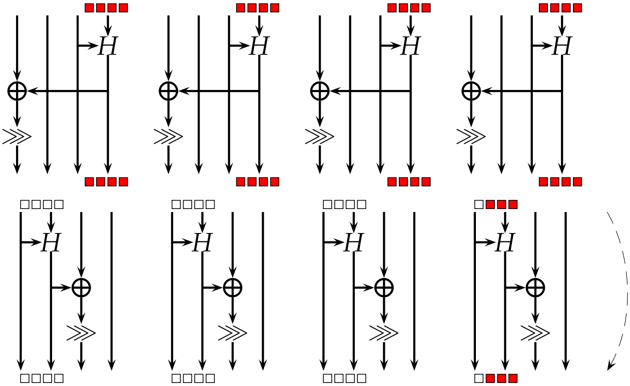

$\tilde{P}_{i+1} = \tilde{p}^0[0:2]$

if $P_1 P_2 \ldots P_i \tilde{P}_{i+1} B_{n-i-1} \geq \bar{B}_n$ assign next bit

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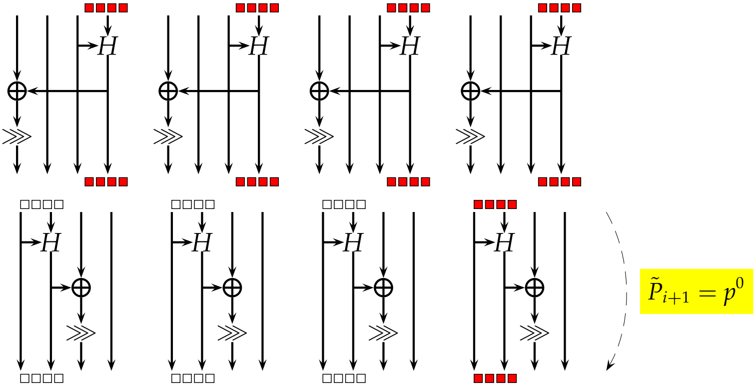

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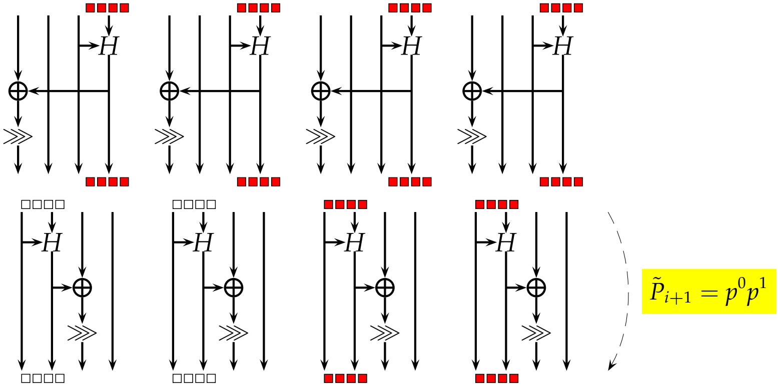

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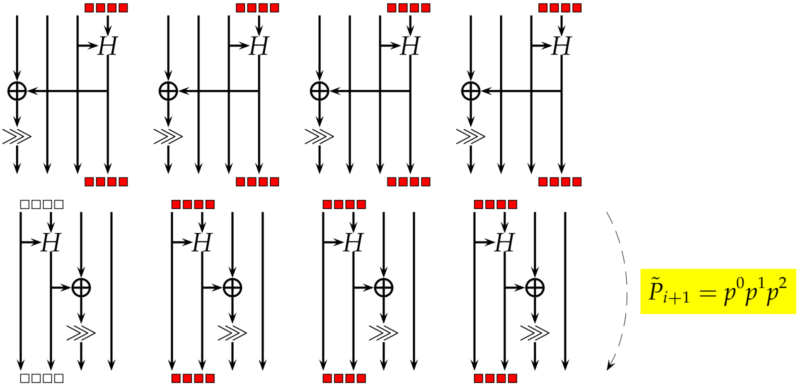

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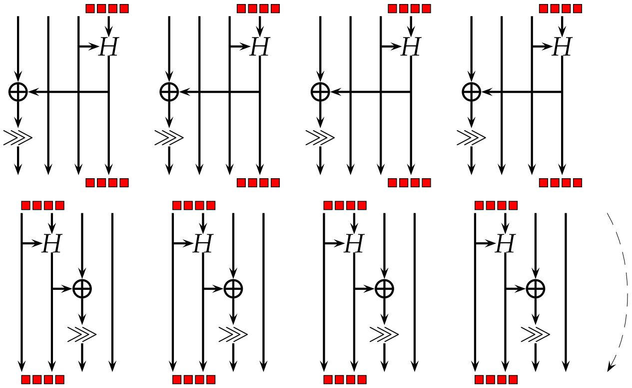


$P_{i+1} = p^0 p^1 p^2 p^3$

if $P_1 P_2 \ldots P_i P_{i+1} B_{n-i-1} \geq \bar{B}_n$ assign next bit

Complexity of BS

Time and Memory Requirements

- Negligible memory (good)
- Time is proportional to num. of rounds and word size (bad)
- Feasible up to $F^{0.5}/F^{1.0}$ rounds (SAT-solver covers up to $F^{1.0}/F^{1.5}$)

Time Measurements for NORX32 and NORX64

- **BS** $F^{0.5}/F^{1.0} \approx$ 49 days; negl. RAM (3.4GHz PC)
- **SAT** $F^{1.0}/F^{1.5} \approx$ 8 hours; 49 GB RAM [AJN14]

Heuristic Search (HS)

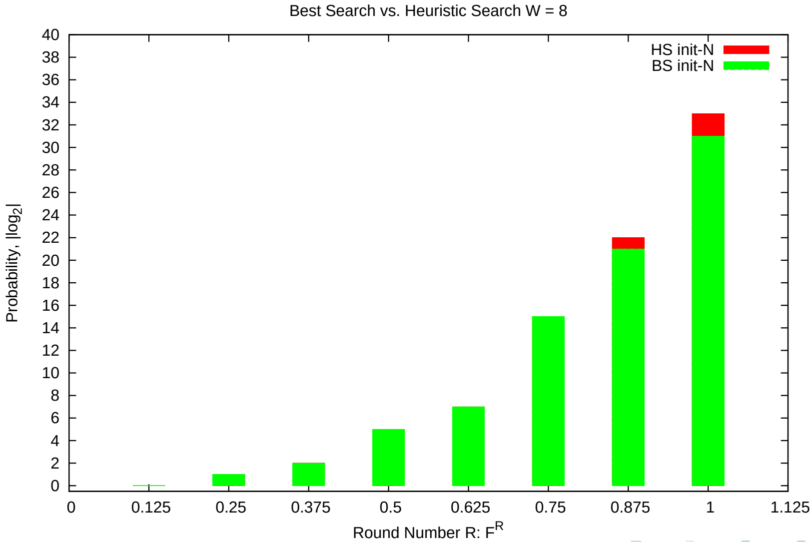
Heuristics to lower the time complexity of BS

1. Time Limit (TL)
   - How long to wait before the recursive call is terminated
2. Maximum Number of Tries (MT)
   - Number of times that we restart the search after the TL is exceeded.
3. Branch Factor Percentage x (BF)
   - In x out of 100 cases branch and explore both 0 and 1 for a given bit. In the remaining cases set the bit to 0.
   - Intuition: due to $\mathrm{xdp}^{\mathrm{H}}(\alpha, \beta \rightarrow \gamma) = 2^{-\mathrm{hw}((\alpha \vee \beta) \ll 1)}$

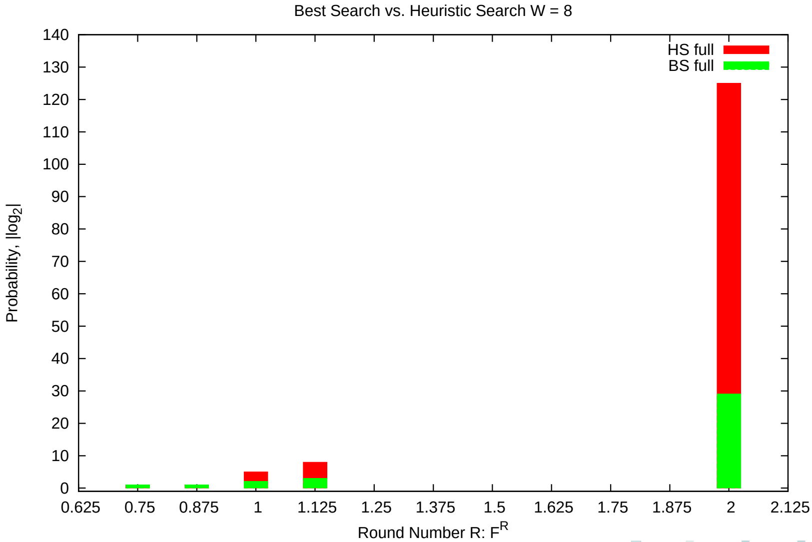
TL, sec	1	1	1	30	60	60	**600**	3600
MT	65535	1024	512	256	256	256	**64**	1
BF	30	50	75	50	50	50	**75**	100

Outline

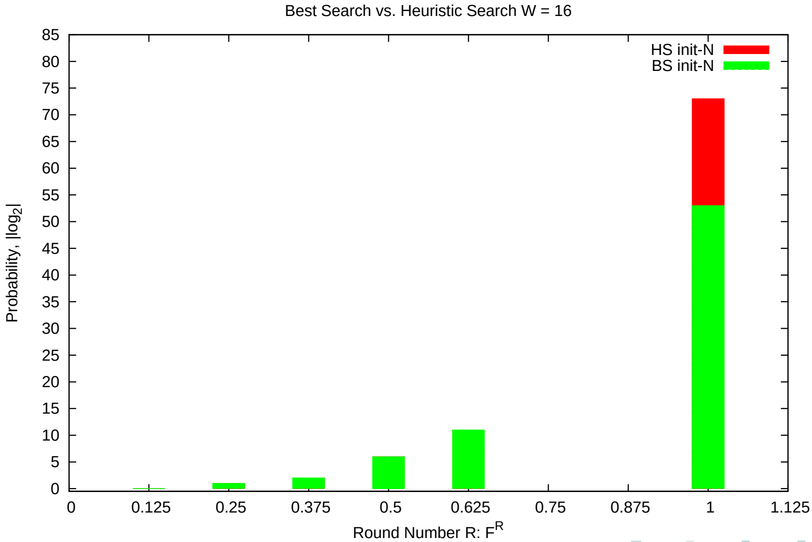
NORX8: BS vs. HS, scenario: init$_N$



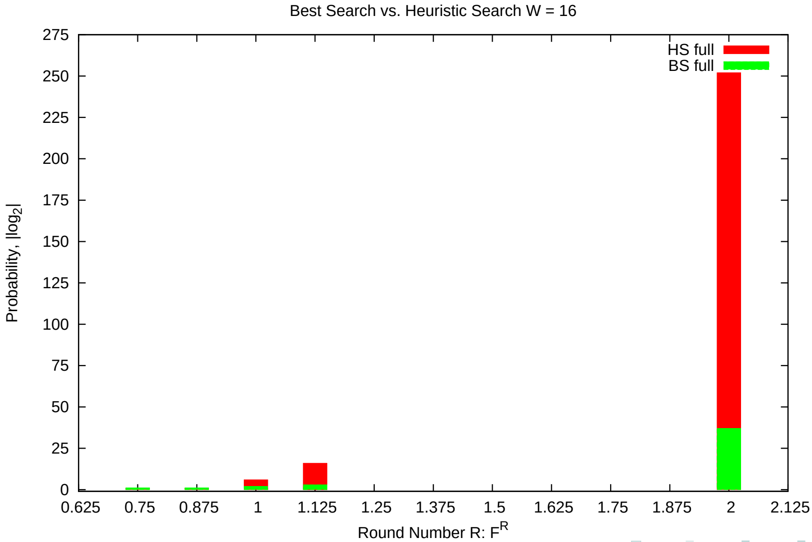
NORX8: BS vs. HS, scenario: full



NORX16: BS vs. HS, scenario: init$_N$



NORX16: BS vs. HS, scenario: full



OUTLINE

Conclusions and Future Work

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- New heuristic algorithm for diff. search in NORX
- Matsui-like; Negligible memory; Manageable time
- Accuracy drastically degrades in the num. rounds and word size

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- New trails on up to $F^{2.0}$ rounds $\implies$ new diff. bounds
- Designers' bounds are too pessimistic (designer's PoV)
- Our bounds are too optimistic (designer's PoV)
- Need better heuristics; new (algorithmic) optimizations; BS + SAT

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Main Message

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Main Message

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Thank you for your attention!

Questions?

References I

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